ROUND I: Graphing on the number line - inequalities, absolute value
DRAW THE GRAPH FOR EACH PROBLEM ON THE NUMBER LINE PROVIDED. SPECIFY ANY NON-INTEGER ENDPOINTS. USE NOTATION LIKE THIS FOR YOUR GRAPHS:
3.5


1. $300-200 \mathrm{x}<500$
2. $7-3|4 y-7| \geq 4$
3. $1<|z| \leq 3$

ANSWERS

(2 pts)

(3 pts) 3


Algonquin, Quaboag, Shrewsbury

## ROUND II: Set theory

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Two sets of people combined total 200. O f these 200, 145 belong to both sets. One of the sets has 170 people in it. How many people are in the other set?
2. A set G consists of all possible elements of the form $\{a, b, c\}$ where $a \in\{0,4,8\}$, $b \in\{w, x, y, z\}$ and $c \in\{\beta, \varphi, \theta, \mu, \chi\}$. How many elements are in set G ?
3. In a recent survey, 3 people used all of soap, fabric softener, and bleach, 6 people used only soap and softener, 12 used only soap, 10 used only softener, 11 used only bleach, the same number of people used just soap and bleach as used just softener and bleach, and 27 of the 75 people surveyed never did laundry. How many of those surveyed used softener and bleach only?

## ANSWERS

(1 pt) 1. $\qquad$
(2 pts) 2. $\qquad$
(3 pts) 3. $\qquad$
Doherty, South, Westborough

## ROUND III: Operations on numerical fractions, decimals, percents, and percentage word problems

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. The population of Austin, Texas increased by $48.89 \%$ between 1970 and 1980 . If the popualation was 536,700 in 1980, to the nearest whole number, what was the population in 1970?
2. Find the number $1 / 3$ of the way from $3 / 7$ to $12 / 13$.
3. The height of a staircase is between 4 and 6 meters. I climb half the steps, then a third of the remaining steps, and finally an eighth of the rest and stop to rest. Each step is 20 cm in height. What is the total height of the staircase, in meters?

## ANSWERS

(1 pt) 1.
(2 pts) 2. $\qquad$
(3 pts) 3.
$m$

Doherty, Shrewsbury, unidentified

ROUND IV: Techniques of counting and probability

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. A nickel, a dime, and a quarter are tossed once each. Find the probability of getting all tails.
2. Rearranging the digits of the number 579 produces different 3 -digit numbers. What is the sum of all such numbers, including 579?
3. There were 4 parts to Terry's most recent test. On part 1 , he had to answer any 2 of 5 questions. On part 2, he had to answer any 3 of 7 questions. On part 3 it was 1 of 3 and on part 4 it was 4 of 9 . How many choices did he have if he had to answer all 10 questions?

ANSWERS
( 1 pt ) 1 . $\qquad$
(2 pts) 2. $\qquad$
(3 pts) 3.
Assabet Valley, Hudson, Southbridge

TEAM ROUND: Topics of previous rounds and open

## ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ROUND ANSWER SHEET

1. Graph the set of just the integers that satisfy $4-|2 x-5| \geq x$. You must also supply relevant coordinates.
2. On the Venn diagram like this on the answer sheet shade $\overline{(\bar{D} \cup[D \cap E]}$, where $\bar{S}$ denotes the complement of set $S$.

3. A tank at a marine exhibit contains 2000 gallons of sea water. The sea water is $7.5 \%$ salt. To the nearest gallon, how many gallons of fresh water must be added to the tank so that the mixture is $7 \%$ salt? Assume that the tank has room for the extra water.
4. In a single toss of two ordinary dice, what is the probability that neither doubles (two of the same number) nor a 5 will appear? Express your answer as a reduced fraction.
5. Find the sum of the prime factors of 2010 and the first prime year in the new millenium.

6. Place each of the integers 3 through 9 in one of these circles in such a way that the sum of the numbers in each set of three collinear circles is 15 .

If the area of parallelogram $A B C D$ is 64 and length $\mathrm{AC}=8$, find length DE if $\overline{D E} \perp \overline{A C}$.

8. If the volume of a cube is doubled in forming a larger cube, by what percent is each edge of the cube increased? Get the nearest $0.01 \%$.

Algonquin, Auburn, Bancroft, Bartlett, Holy Name, Shrewsbury, Westborough, Worcester Academy

ROUND I (l ot) 1 .
 \# line
graphs
$(2 n+s)$ .

(3 bots) 3.


Ruin II (1 nt) 1. 175 sets

$$
\begin{array}{lll}
(2 \text { ts }) & 2 & 60 \\
(3 \text { ts }) & 3 & 3
\end{array}
$$

ROUND III
(1 nt)

1. 360,467
fr , dec

$$
(2 n t s) \quad 2 . \quad \frac{54}{91}
$$

$(3$ orts $) 3.4 .8$ o $4 \frac{4}{5} \mathrm{~m}$

R(TJN) IV (I nt) $1 . \frac{1}{8}$ a .125 a $12.5 \%$ counting. prob (2nts) 2. 4662
( pts) 3. 132,300

TEAM ROUND $\begin{aligned} & 3 \text { pots } \\ & \text { each }\end{aligned}$
1.

2.

3. 143 gal
4. $\quad \frac{5}{9}$
5. 2080

25.997

Round I

$$
\begin{aligned}
3-2 x & <5 \\
-2 x & <2 \\
x & >-1
\end{aligned}
$$

2. 

$$
\begin{aligned}
-3|4 y-7| & \geq-3 \\
|4 y-7| & \leq 1
\end{aligned}
$$

$4 y-7 \leq 1$ and $4 y-7 \geq-1$
$y \leq 2$ and $y \geq \frac{3}{2} \Rightarrow \frac{3}{2} \leq y \leq 2$
3. $1<z \leq 3$ or $-3 \leq z<1$ by nature of abs value

Round II
1.


$$
\begin{aligned}
170+x & =200 \\
x & =30
\end{aligned}
$$

Other set

$$
145+30=175
$$

2. (3 possible items for a) times (4.. .. " 1 ) times

$$
(5 \cdot \cdot \cdot c)=60
$$

3. 



$$
\begin{gathered}
3+6+12+10+11+2 x=75 \\
2 x=6 \Rightarrow x=3
\end{gathered}
$$

ROUND III

1. $1980 \mathrm{\rho up}=148.897$ of $1970 \mathrm{\rho op}$

$$
536,700=1.4889 x
$$

nearest integer $x=360,467$

ROUND III cont.
2. $\frac{12}{13}-\frac{3}{7}=\frac{84-39}{91}=\frac{45}{91}$

$$
\begin{equation*}
\frac{3}{7}+\frac{1}{3}\left(\frac{45}{91}\right)=\frac{3}{7}+\frac{15}{91}= \tag{54}
\end{equation*}
$$

OR weighted average

$$
\frac{2}{3}\left(\frac{3}{7}\right)+\frac{1}{3} \cdot\left(\frac{12}{13}\right)=\cdots=\frac{54}{91}
$$

3. $\frac{1}{2}+\frac{1}{3}\left(\frac{1}{2}\right)+\frac{1}{8}\left(\frac{1}{3}\right)=\frac{12+4+1}{24}$

$=\frac{17}{24}=$ fraction of total ht
and when multiplied by the number of steps gives the number af steps climbed, an integer.
If there are 24 steps, each 0.2 m high, total ht $=24(.2) € 4.8 \mathrm{~m}$ with 48 or more steps. the total ht is too big

Round IV

1. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}=\frac{1}{8}$
2. $\begin{array}{r}579 \\ 597 \\ 754 \\ 795 \\ 957 \\ 975 \\ \hline 4662\end{array}$
$3 \quad{ }_{5} C_{2} \cdot{ }_{7} C_{3} \cdot{ }_{3} C_{1} \cdot{ }_{9} C_{4}$

$$
\begin{aligned}
& =1035 \cdot 3 \cdot 126 \\
& =132,300
\end{aligned}
$$

$$
\left\langle\begin{array}{l}
\text { wist } \frac{1}{2} \\
<_{T<} \leq \\
\text { on each } \\
\text { branch }
\end{array}\right.
$$

Lan 12. iOU WOCOMAL Freshman Meet BRIEF SOLUTIONS cont.
team round

1. If $2 x \geq 5,4-2 x+5 \geq x$

$$
\begin{aligned}
\left(x>2 \frac{1}{2}\right) & 9 \geq 3 x \sim x \leq 3 \\
& (x=3, \text { enl y }
\end{aligned}
$$

If $2 x<5,4+2 x-5 \geq x$ $\left(x<2 \frac{1}{2}\right) \quad x \geq 1 \quad x=1,2$
2. DME

$$
\overline{0} \cup(D \cap E)
$$

Complement of this is
3 orig salt $=$ final salt $x=$ gal fresh water to add

$$
\begin{gathered}
.075(2000)=.07(2000+x) \\
150=140+.07 x \\
x=143 \text { nearest gal. }
\end{gathered}
$$

4. $6 \cdot 6=36$ equally likely outcomes. 5 passible first number ( $1,2,3,4,6$.) and 4 possible second numbers. $5.4=20$ and prosaic. $=\frac{20}{36}=\frac{5}{9}$
5. $2010=10 \cdot 201=\underbrace{5.2 \cdot 3.67}_{\operatorname{sim} 77}$

First prime year is 2003

$$
77+2003=2080
$$

6. $\triangle$ area $=\frac{1}{2} A C \cdot D E=\frac{1}{2} \square$ area

$$
\begin{gathered}
4 \cdot D E=32 \\
D E=8
\end{gathered}
$$

$25, \frac{1}{3}$ of 15 , must 90 in the central circle. Then pairs of opposites must have sum 10
8. Since 70 change is sought. suppers cubes have volumes 1 and 2. Then edger are (and $\sqrt[3]{2}$.

$$
\begin{aligned}
& \sqrt[3]{2}=\frac{x}{100}(1) \\
& 125.99=x
\end{aligned}
$$

$\sqrt[3]{2}$ is $125.99 \%$ of 1 Increase is $25.99 \%$

